DSN Progress Report 42-52 May and June 1979

Internal Noise of a Phase-Locked Receiver With a Loop-Controlled Synthesizer

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We propose a local oscillator design that uses a digitally programmed frequency synthesizer instead of an analog VCO. The integral of the synthesizer input, the "digital phase," is a convenient measure of integrated doppler. We examine the internal noise of such a receiver. At high carrier margin, the local oscillator phase noise equals that of the Block IV receiver, about 2 deg rms at S-band, whereas the digital phase noise is about 0.5 deg rms.

I. Introduction

A simplified diagram of the carrier tracking function of the Block IV receiver is shown in Fig. 1. To obtain the local oscillator output, the doubled output of the voltage-controlled crystal oscillator (VCXO) is mixed with the output of the Dana Synthesizer and multiplied up to the first heterodyne frequency at S-band or X-band. The synthesizer is programmed to follow the previously estimated carrier frequency, and the difference between the estimated and actual carrier phases is made up by the loop-controlled VCXO.

It has been proposed that the VCXO be replaced by a loop-controlled digital frequency synthesizer such as the Dana Synthesizer already used in the programmed oscillator. This idea leads to the local oscillator design shown in Fig. 2. The 50-MHz reference clocks the synthesizer and is also mixed with the synthesizer output; the purpose of mixing here is to reduce the factor by which synthesizer phase noise is multiplied. The gain applied to the digital loop error signal is chosen to make the loop transfer function essentially the same as that of the present Block IV receiver, for a chosen bandwidth setting.

An important feature is the *digital phase*. At present, doppler phase is extracted by mixing the local oscillator output with a signal from the exciter. In the proposed design, the frequency programmed into the synthesizer is integrated digitally to produce a phase that is in step with the contents of the synthesizer's internal phase register. The synthesizer itself is a phase-locked loop that tries to keep its output phase in step with this internal register. Thus the digital phase can be expected to stay close to the local oscillator phase.

The main purpose of this article is to compare the digital phase noise of the proposed receiver to the local oscillator phase noise of the present Block IV receiver. The models include only the "internal" phase noises due to the Block IV VCXO and the Dana Synthesizers; in particular, thermal noise and the jitter of the station reference timing signal are ignored. Effects of sampling and quantization in the proposed receiver are also ignored.

We shall see that, for high carrier margin (at least 40 dB above threshold), most of the noise in the Block IV local oscillator comes from the Dana Synthesizer. Thus, the local

oscillator phase noises of the present and proposed receivers are almost equal, about 2 deg rms at S-band. The rms noise of the digital phase of the proposed receiver is about 0.5 deg, but grows very slowly as integration time increases.

II. Properties of the Dana Synthesizer

Since the Dana Synthesizer appears in both the present and the proposed local oscillators, we shall first set down our best estimates of its transfer function and phase noise spectrum. Assume that the reference timing signal has no jitter. Let $\nu(t)$ be the frequency programmed into the synthesizer. (Actually, $\nu(t)$ is sampled every 10 μ s but we shall treat time as continuous.) Then the synthesizer output phase is

$$2\pi \frac{L_d(s)}{s} \nu(t) + n_d(t)$$

where $L_d(s)$ is the phase-to-phase transfer function of the device, and $n_d(t)$ is the additive phase noise.

A gross approximation of the transfer function is given by

$$L_d(s) = \frac{1 + t_0 s}{1 + t_0 s + t_0^2 s^2 / 2} , \qquad (1)$$

where $t_0 = 165 \mu s$. The one-sided fiducial bandwidth of the filter $L_d(s)$ is $0.75/t_0 = 4500 \text{ Hz}$, which is large enough so that we may replace $L_d(s)$ by 1 when using the synthesizer as a loop-controlled oscillator in a narrow-band tracking loop.

Our estimate of the one-sided spectral density of the phase noise n_d in rad²/Hz is given by

$$S_d(2\pi f) = \frac{N_d}{f}$$
, $0 < f \le 100 \text{ Hz}$
 $= \frac{N_d}{100}$, $100 \text{ Hz} \le f \le 10 \text{ Hz}$
 $= \frac{N_d 10^{10}}{f^3}$, $10 \text{ kHz} \le f \le 50 \text{ kHz}$
 $= 0$, $f > 50 \text{ kHz}$ (2)

where

$$N_d = 1.5 \times 10^{-8}$$
.

This noise is nonstationary because of the 1/f term at low frequencies. The noise variance for Fourier frequencies f above 1 Hz is

$$\sigma_d^2 = 2.3 \times 10^{-6} \text{ rad}^2.$$
 (3)

Jitter in the reference is not included. Both N_d and σ_d^2 have a ± 3 dB uncertainty.

The following explains how these values were arrived at; the uninterested reader can skip to Section III.

We first carried out a theoretical computation of the Dana Synthesizer phase noise spectrum. The model consisted of a VCO with $1/f^3$ phase noise spectrum (Ref. 1), controlled by a second-order loop with perfect integrator. This yielded a synthesizer phase noise spectral density that behaves like f at low frequencies. Because two references (Refs. 1 and 2) report spectral densities approximately like 1/f at low frequencies, we were forced to abandon our model and rely wholly on an understanding of the synthesizer as a black box.

The shape of the spectrum in (2) comes from a paper by G. Gillette of Dana Laboratories (Ref. 1); he plots the spectral density down to f=1 Hz. Meyer and Sward of JPL (Ref. 2) measured a spectral density like $1/f^{0.75}$ for 4 Hz $\leq f \leq 80$ Hz. We shall assume a 1/f behavior down to zero frequency. This assumption may be dangerous.

Because a frequency synthesizer derives its time base from a reference oscillator, whether internal or external, one must enquire whether published reports of synthesizer phase noise include the jitter of the reference. Meyer and Sward used a technique that cancels reference jitter. Dana's manual for its synthesizer (Ref. 3) gives a method for measuring total phase noise variance (discussed below); this method also cancels reference jitter. We therefore assume that Gillette's graph does not include reference phase noise.

Since Refs. 1 and 2 deal with an old model of the Dana Synthesizer, we used Gillette's graph to get the shape of the phase noise spectrum, but used phase noise specifications of a current model to obtain the normalizing constant N_d . Reference 3 specifies a total phase noise of -54 dB = 4 \times 10⁻⁶ rad² in the band f_0 ±15 kHz, excluding f_0 ±1 Hz, where f_0 is the programmed output frequency. Figure 3 shows in simplified form the setup used to verify this specification. The reference signal of synthesizer 2 is slaved to that of synthesizer 1.

Suppose that the synthesizers have outputs

$$x_1(t) = \cos(2\pi f_1 t + \theta_1(t) + \theta_0)$$

$$x_2(t) = \cos(2\pi f_2 t + \theta_2(t)).$$

where f_1 and f_2 are the programmed frequencies, $\theta_1(t)$ and $\theta_2(t)$ are independent, zero-mean phase noises, and θ_0 is a constant phase offset. The rms voltmeter sees the signal

$$y(t) = K \sqrt{2} \cos(2\pi (f_1 - f_2) t + \theta(t) + \theta_0),$$

where $\theta(t) = \theta_1(t) - \theta_2(t)$, and K is an unknown constant. The first thing we do in the test is to set $f_1 = f_2 + 1$ kHz. Then the voltmeter reading is K. This is the reference level. Next, we set $f_1 = f_2 + f$, where 0.05 Hz $\leq f \leq$ 0.1 Hz, and observe the peak reading of the rms voltmeter, which now sees the signal

$$y(t) = K \sqrt{2} \left[\cos \left(2\pi f t + \theta_0 \right) \cos \theta(t) \right]$$

$$-\sin(2\pi ft + \theta_0)\sin\theta(t)$$
].

The phase noise specification is defined to be the square of the ratio of peak reading to reference level. Over the short run, say up to one minute, we can assume $\theta(t) \ll 1$ rad; hence

$$y(t) \approx K \sqrt{2} \left[\cos \left(2\pi f t + \theta_0 \right) - \theta(t) \sin \left(2\pi f t + \theta_0 \right) \right].$$

The voltmeter, insensitive below 1 Hz (Fig. 3), does not respond to the first term. The second term causes the voltmeter reading to rise and fall every 1/(2f) seconds; the peak reading, $K\sqrt{2}\sigma(\theta)$, occurs when the synthesizers are in quadrature. Dividing by the reference level K, we conclude from the published phase noise specification that

$$2\sigma^2(\theta) = 2(\sigma^2(\theta_1) + \sigma^2(\theta_2)) = 4 \times 10^{-6} \text{ rad}^2.$$

Therefore, the larger of $\sigma^2(\theta_1)$ and $\sigma^2(\theta_2)$ is between 10^{-6} rad² and 2×10^{-6} rad². We shall use

$$\sigma_{\rm spec}^2 = 2 \times 10^{-6} \, \text{rad}^2$$
.

To compute N_d we set

$$\int_{1}^{15000} S_d(2\pi f) \, df = \sigma_{\text{spec}}^2.$$

Integrating (2) gives $\sigma_{\text{spec}}^2 = 131.4 N_d$, whence $N_d = 1.5 \times 10^{-8}$.

A more direct estimate of N_d comes from the single side band (SSB) phase noise spectral density specification in Ref. 3. The phase noise in the band f_0 ±15 kHz, excluding f_0 ±10 Hz, is given as -90 dB/Hz maximum. The SSB spectral density, denoted by $\mathscr{L}(f)$ in the literature, is asymptotically related to the baseband phase noise spectral density $S_d(2\pi f)$ by

$$\mathscr{L}(f) \sim \frac{1}{2} \, S_{d}(2\pi f),$$

provided f is large enough so that

$$\int_f^\infty S_d(2\pi f') \, df' \ll 1 \, \mathrm{rad}^2$$

(Ref. 4). From (2) we set $\frac{1}{2}S_d(20\pi) = 10^{-9}$. Then $N_d/20 = 10^{-9}$; $N_d = 2 \times 10^{-8}$. This agrees well with the 1.5×10^{-8} computed earlier. We settle on the latter figure with a ± 3 -dB uncertainty.

Finally, the value of σ_d^2 in (3) is obtained from $\sigma_{\rm spec}^2$ by including the extra power from 15 kHz to 50 kHz.

III. Internal Noise of Present Block IV Receiver

The rms internal local oscillator phase noise at S-band is given in Table 1 as a function of design point bandwidth b_{LO} and carrier margin. The total noise is the RSS of the noises from the VCXO and the Dana Synthesizer. At high carrier margin, most of the noise, about 1.7 deg, comes from the Dana Synthesizer and is almost independent of bandwidth. Most of the local oscillator noise is outside the loop passband. Derivations of these results follow.

From Fig. 1 we see that the noise in the local oscillator phase $\hat{\theta}$, with the VCXO input shorted, is

$$M\left(n_{d}(t) + 2n_{v}(t)\right)$$
,

where n_d and n_v are the phase noises of the Dana Synthesizer and the VCXO, and M=20 for S-band. The noise n_d is described in Section II. We shall assume that n_v has a one-sided spectral density N_v/f^3 ; the constant N_v will be calibrated from receiver specifications.

When the phase-locked loop is allowed to control the local oscillator, its phase noise becomes

$$(1 - L(s)) M(n_d(t) + 2n_v(t)),$$

where

$$L(s) = \frac{1 + \tau_2 s}{1 + \tau_2 s + \tau_2^2 s^2 / r},$$
 (4)

$$\tau_2 = 0.75/b_{LO}$$

and r depends on the one-sided threshold loop bandwidth b_{LO} and on the carrier margin. At threshold, r=2. At high margin,

$$r = 32 \text{ for } 2b_{LQ} = 1, 10, 100 \text{ Hz}$$

=
$$18.5$$
 for $2b_{LQ} = 3,30,300$ Hz

We are using the perfect integrator approximation of the loop filter, $F(s) = (1 + \tau_2 s)/(\tau_1 s)$.

The high-pass filter 1 - L(s), acting on n_d , cancels the 1/f spectrum at low frequencies and passes most of the rest. Therefore, the rms contribution of the Dana Synthesizer to local oscillator phase noise is essentially

$$M\sigma_d = 3.0 \times 10^{-2} \text{ rad} = 1.7 \text{ deg}$$
.

We shall ignore the decrease at the higher loop bandwidths since it is at most about 0.1 deg.

The variance σ_v^2 of $(1 - L(s)) n_v$ is given on p. 32 of Ref. 5. It is convenient to recast those formulas as follows:

$$\sigma_{v}^{2} = \frac{N_{v} \tau_{2}^{2}}{r^{2} \sqrt{-D}} \operatorname{Sin}^{-1} \left(\frac{r \sqrt{-D}}{2}\right), \qquad 2 \leq r < 4$$

$$= \frac{N_{v} \tau_{2}^{2}}{8}, \qquad r = 4$$

$$= \frac{N_{v} \tau_{2}^{2}}{r^{2} \sqrt{D}} \ln \left(\frac{r}{2} - 1 + \frac{r \sqrt{D}}{2}\right), \quad r > 4$$

where D = 1 - 4/r.

The specifications of the Block IV Receiver-Exciter Subsystem give a rms phase jitter of 9 deg when $\tau_2=1.5$ s, r=2, M=20. (This results from operating the VCXO in a phase-locked loop with these parameters and a strong signal, so that thermal noise is absent.) This calibrates N_{ν} and allows us to fill in the VCXO noise column of Table 1, which lists $2M\sigma_{\nu}$ at threshold and at high margin. When the receiver is switched to wide mode, N_{ν} is multiplied by 100, τ_2 is divided by 10, and r is unchanged. Therefore, b_{LO} is multiplied by 10 and σ_{ν} is unchanged.

Table 1 also lists the total rms internal noise $\sigma(\widehat{\theta})$, where

$$\sigma^2(\widehat{\theta}) = (M\sigma_d)^2 + (2M\sigma_v)^2.$$

IV. Internal Noise of Proposed Receiver

We now have to consider two phases, the local oscillator output phase $\widehat{\theta}$ and the digital phase $\widetilde{\theta}$. The rms noise $\sigma(\widehat{\theta})$ is about 1.7 deg at S-band, the same as the Dana Synthesizer component of the noise in the present Block IV receiver. Since the VCXO is absent, $\sigma(\widehat{\theta})$ does not rise as carrier margin decreases. Again, most of the $\widehat{\theta}$ noise is outside the loop passband.

On the other hand, most of the noise in the digital phase $\widetilde{\theta}$ is inside the loop passband. This noise turns out to be nonstationary, with a spectral density like 1/f at low frequencies. Thus it makes no sense to quote a single variance $\sigma^2(\widetilde{\theta})$. Instead, we compute

$$\sigma^{2}(\widetilde{\theta}, T) = \frac{1}{2} \operatorname{var} \left[\widetilde{\theta}(t+T) - \widetilde{\theta}(t)\right]$$
$$= \frac{1}{2} \operatorname{var} \int_{t}^{t+T} \frac{d\widetilde{\theta}}{dt} dt,$$

which is independent of t because $d\widehat{\theta}/dt$ is stationary. The factor 1/2 is needed to make a fair comparison with $\sigma(\widehat{\theta})$, for if we apply the operator σ^2 (*, T)to the *stationary* process $\widehat{\theta}$, we get

$$\sigma^{2}(\widehat{\theta}, T) = \frac{1}{2} \operatorname{var}(\widehat{\theta}(t+T) - \widehat{\theta}(T))$$

$$= R(0) - R(T)$$

$$\to R(0) = \sigma^{2}(\widehat{\theta}) \quad \text{as } T \to \infty,$$

where R is the autocovariance function of $\widehat{\theta}$. On the other hand, we shall see that

$$\sigma^2(\widetilde{\theta}, T) \to \infty$$
 as $T \to \infty$.

Table 2 lists the rms noise $\sigma(\widetilde{\theta}, T)$ at S-band as a function of T/τ_2 and the loop threshold bandwidth b_{LO} , where $\tau_2 = 0.75/b_{LO}$, for high carrier margin. The values decrease slightly as margin decreases. Since $\widetilde{\theta}$ is "almost" stationary, $\sigma^2(\widetilde{\theta}, T)$ grows slowly with T, in fact, like log T. We must warn the reader, however, that this result holds only if the spectral density $S_d(2\pi f)$ of the Dana Synthesizer phase noise behaves like 1/f at least down to f = 1/(4T). Derivations of these results follow.

Although the loop error signal of the proposed receiver is digital, we shall proceed with an analog analysis. A phase-locked receiver using the local oscillator of Fig. 2 has the closed-loop transfer function.

$$L_1(s) = \frac{1 + \tau_2 s}{1 + \tau_2 s + \tau_2^2 s^2 / (r L_d(s))}$$

where $L_d(s)$, given by (1), is the transfer function of the Dana Synthesizer. The one-sided bandwidth of $L_d(s)$ is about 4500 Hz. The internal noise in the local oscillator phase $\widehat{\theta}$ is

$$(1 - L_1(s)) Mn_d(t)$$
 (5)

where $n_d(t)$ is the Dana Synthesizer phase noise. As with the Block IV receiver, the rms value $\sigma(\hat{\theta})$ is approximately $M\sigma_d = 1.7$ deg for S-band.

To obtain the internal noise in the digital phase $\tilde{\theta}$, suppose that $n_d(t)$ is the only active influence on the loop. The frequency (Hz) programmed into the synthesizer is $(1/2\pi M)$ $d\tilde{\theta}/dt$. The local oscillator phase (rad) is

$$\widehat{\theta} = M \left(2\pi \frac{L_d(s)}{s} \frac{s\widetilde{\theta}}{2\pi M} + n_d \right)$$

$$= L_d(s)\widetilde{\theta} + Mn_d.$$

Replacing $\hat{\theta}$ by (5), we get

$$\widetilde{\theta} = \frac{\widehat{\theta} - Mn_d}{L_d(s)} = -\frac{L_1(s)}{L_d(s)} Mn_d$$
.

For computation, we shall approximate L_1/L_d by the Block IV transfer function L, given by (4). Then

$$\sigma^2(\widetilde{\theta}, T) = \frac{1}{2} \operatorname{var} \int_0^T \frac{d\widetilde{\theta}}{dt} dt$$

$$= \int_0^\infty (1 - \cos \omega T) S_{\widetilde{\theta}}(\omega) \frac{d\omega}{2\pi}$$

$$= M^2 \int_0^\infty (1 - \cos \omega T) |L(i\omega)|^2 S_d(\omega) \frac{d\omega}{2\pi}$$

where $S_{a}(\omega)$ is given by (2). This leads to the further approximation

$$\sigma^2(\widetilde{\theta},\,T)\,=\,M^2N_d\,(J_1^{}+J_2^{})$$

where

$$J_1 = \int_0^\infty (1 - \cos \omega T) |L(i\omega)|^2 \frac{d\omega}{\omega}$$
 (6)

is the main portion, and

$$J_{2} = \frac{1}{200\pi} \int_{200\pi}^{\infty} |L(i\omega)|^{2} d\omega$$
 (7)

is a correction that is significant only for $2b_{LO} = 100$ Hz or 300 Hz. In the Appendix, these integrals are evaluated in closed form in terms of the elementary functions and the exponential integral $\int (e^z/z) dz$. Numerical results are given in Table 2.

V. Conclusions

We have proposed a design for a receiver local oscillator containing a loop-controlled synthesizer. The transfer function of the proposed receiver and its performance in the presence of thermal noise are essentially the same as those of the present Block IV receiver. The proposed design provides a convenient digital estimate of carrier phase, obtained by integrating the synthesizer input frequency.

For S-band and high carrier margin, the rms internal phase noises of the present (Block IV) and proposed receivers are approximately as follows:	1.7 deg	Proposed local oscillator phase noise. Insensitive to carrier margin.
	$0.3 - 0.7 \deg$	Proposed digital phase noise.

Appendix

Evaluation of Two Integrals

The integrals in question are J_1 and J_2 , given by (6) and (7). We rewrite them as follows:

$$J_1 = \int_{-\infty}^{\infty} \frac{1 - \cos ax}{x} f(x) dx$$

$$J_2 = \frac{1}{x_0} \int_{x_0}^{\infty} f(x) \, dx$$

where

$$f(x) = \frac{1 + rx^2}{1 + (r - 2)x^2 + x^4},$$

$$a = \sqrt{r} T/\tau_2, x_0 = 200\pi \tau_2/\sqrt{r}.$$

From now on, we assume that r > 4.

The partial fraction expansion of f(x) is

$$f(x) = \frac{A_1}{x^2 + x_1^2} + \frac{A_2}{x^2 + x_2^2}$$

where

$$A_1 = \frac{1 - rx_1^2}{x_2^2 - x_1^2}$$
, $A_2 = \frac{1 - rx_2^2}{x_1^2 - x_2^2}$

$$x_2^2 = \frac{r}{2} - 1 + \frac{r}{2}\sqrt{1 - 4/r}, \ x_1^2 = 1/x_2^2.$$

Integrating f(x), we get

$$J_2 = \frac{x_2}{x_0(x_2^4 - 1)} \left[\left(x_2^2 - r \right) \operatorname{Tan}^{-1} \frac{1}{x_0 x_2} + \left(r x_2^2 - 1 \right) \operatorname{Tan}^{-1} \frac{x_2}{x_0} \right].$$

To evaluate \boldsymbol{J}_1 we evaluate the integral

$$I(t) = \int_0^\infty \frac{1 - \cos tx}{x \left(1 + x^2\right)} dx$$

In terms of the exponential integrals

$$E_1(t) = \int_t^\infty \frac{e^{-x}}{x} \ dx \,,$$

$$Ei(t) = \int_{-\infty}^{t} \frac{e^{x}}{x} dx,$$

which are tabulated in Ref. 6. We use Raabe's integrals (Ref. 7):

$$I_s(t) = \int_0^\infty \frac{\sin tx}{1+x^2} dx = \frac{1}{2} \left[e^t E_1(t) + e^{-t} \operatorname{Ei}(t) \right],$$

$$I_c(t) = \int_0^\infty \frac{x \cos tx}{1 + x^2} dx = \frac{1}{2} \left[e^t E_1(t) - e^{-t} \operatorname{Ei}(t) \right] .$$

(The integral form of $I_c(t)$ will not play a part here.) Since

$$I(t) = \int_0^t I_s(u) du, \qquad (8)$$

$$\frac{d}{dt}I_c(t) = I_s(t) - \frac{1}{t},$$

we have, for some constant C,

$$I(t) = I_{o}(t) + \ln t + C, \qquad t > 0.$$

As $t \to 0$, $I(t) \to 0$ from (8). Limiting expressions for $E_1(t)$ and $E_1(t)$ give

$$I_c(t) + \ln t \rightarrow -\gamma$$
 as $t \rightarrow 0$,

$$J_1 = (1+k)I_c(ax_2) - kI_c(a/x_2) + \gamma + (1+2k)\ln x_2 + \ln a$$

where γ is Euler's constant, 0.577216. Therefore, $C = \gamma$.

where

For large t,

$$I(t) = \ln t + \gamma - 1/t^2 + O(1/t^4).$$

 $k = \frac{2x_2^2 + 1}{x_2^4 - 1} \, .$

Finally, we have

These formulas were spot-checked by numerical integration.

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Table 1. RMS internal noise of Block IV receiver at S-band (total noise is RSS of VCXO noise with Dana synthesizer noise $M\sigma_d=1.7~{\rm deg})$

$^{2b}{}_{LO}, \ _{ m Hz}$		Carrier	VCXO noise,	Total noise, $\sigma(\widehat{\theta})$,
Narrow	Wide	margin	$2M\sigma_{_{\mathcal{V}}},$ deg	deg
1	10	1	9	9
		00	0.9	2.0
3	30	1	3.0	3.5
		00	0.5	1.8
10	100	1	0.9	2.0
		∞	0.09	1.7
30	300	1	0.3	1.8
		∞	0.05	1.7

Table 2. RMS internal noise of digital phase $\widetilde{\theta}$ of proposed receiver at S-band as a function of integration time T (high carrier margin assumed)

		$T/\tau_2,\tau_2$ =	$0.75/b_{LO}$		
		1	10 ²	10 ⁴	10 ⁶
	1	0.29	0.42	0.52	0.60
	3	0.28	0.41	0.51	0.59
$2b_{LO}$,	10	0.29	0.42	0.52	0.60
Hz	30	0.29	0.42	0.52	0.60
	100	0.42	0.52	0.60	0.67
	300	0.50	0.58	0.66	0.72

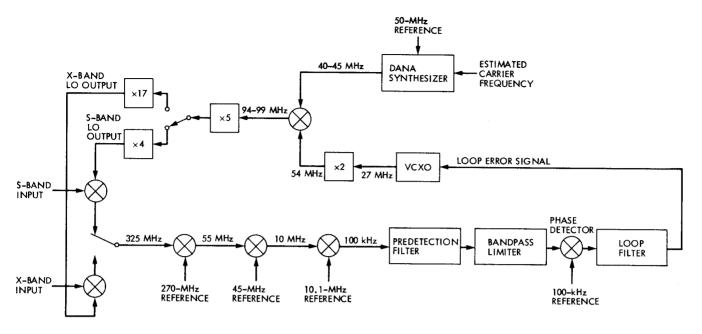


Fig. 1. Block IV carrier tracking receiver

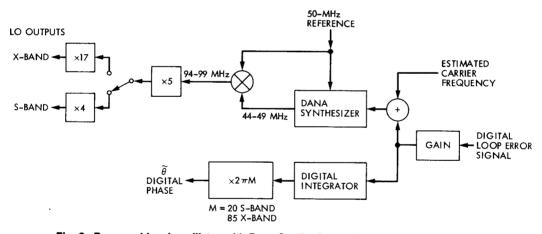


Fig. 2. Proposed local oscillator with Dana Synthesizer as loop-controlled oscillator

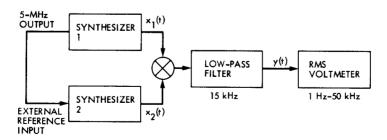


Fig. 3. Measurement of synthesizer phase noise variance